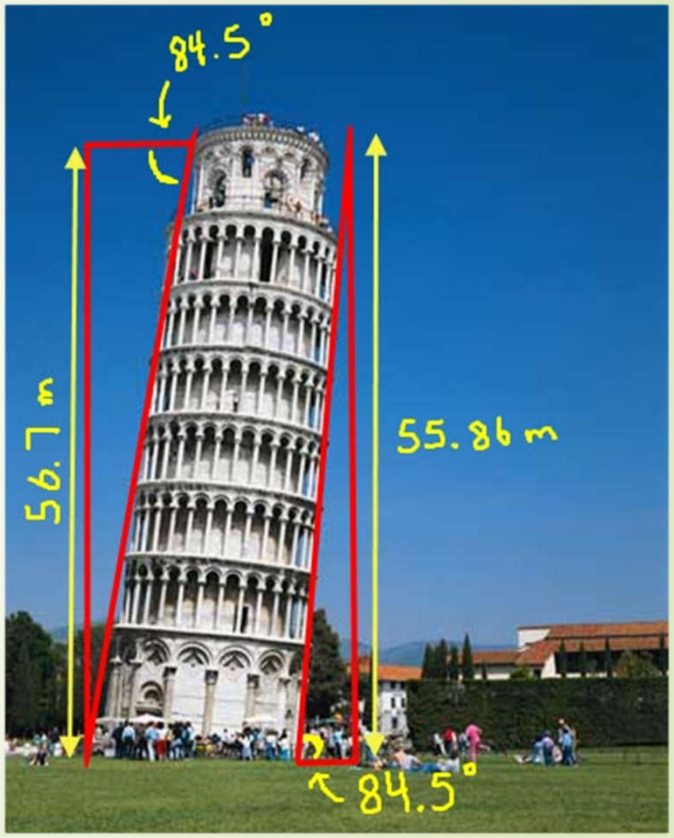
INVERSE

2. Inverse trigonometry

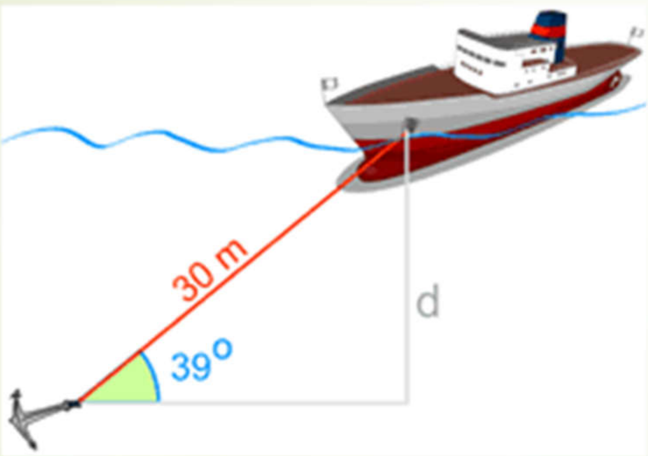
- Introduction
- Basic Concepts
- Inverse trigonometric functions & their Graphs
- Properties of Inverse Trigonometric Functions



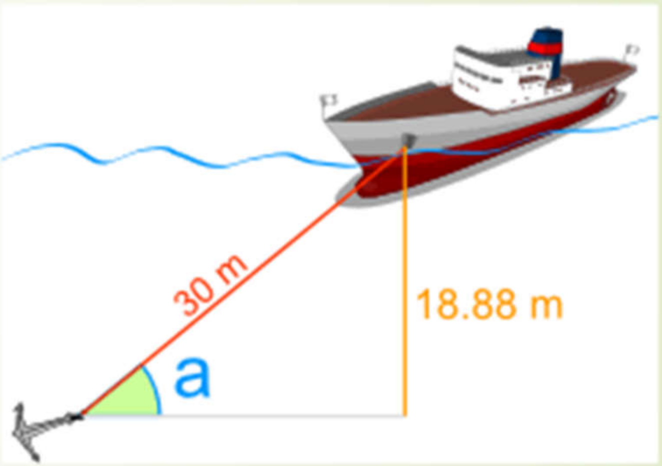
Introduction

- There are real-life situations in which we need to determine the angle, not lengths.

normal



inverse

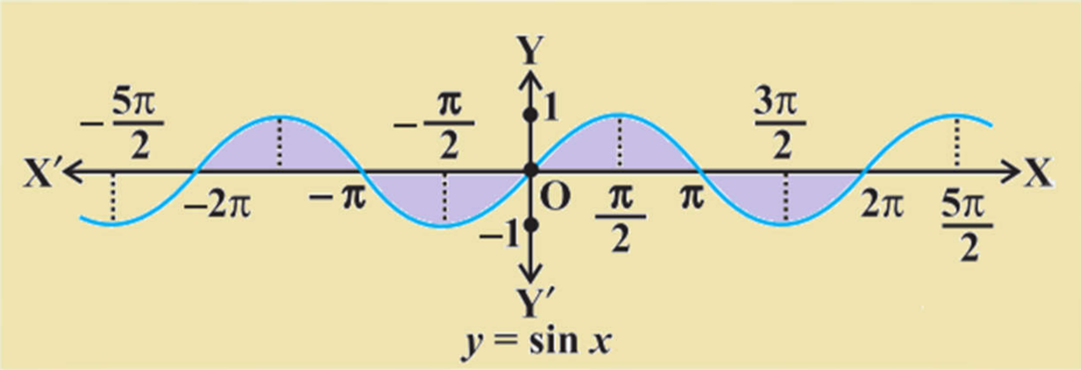


Inverse of functions

- The inverse of a function '*f*' exists if '*f*' is one-one and onto.
- Now, trigonometric functions are not one-one and onto over their natural domains and ranges and hence their inverses do not exist.
- So we shall study about the restrictions on domains and ranges of trigonometric functions and observe their through graphical representations.

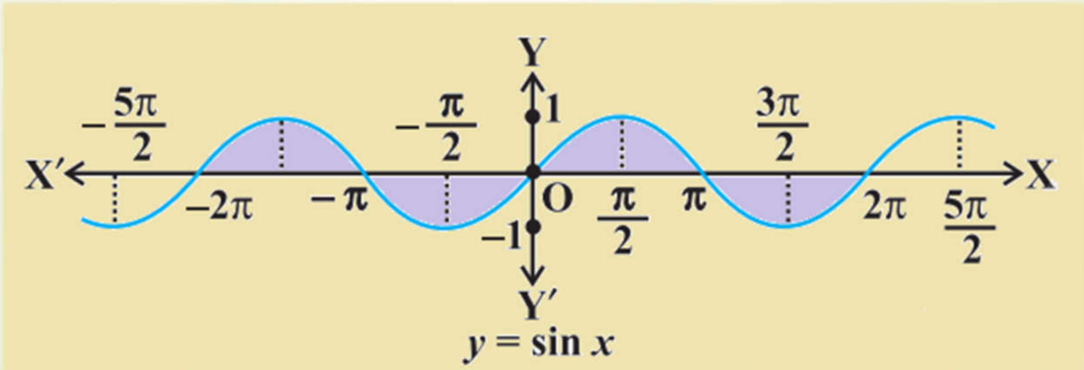
Functions : Natural Domain & range

- Sine function, i.e., sine : $\mathbf{R} \rightarrow [-1, 1]$
- Cosine function, i.e., cos : $\mathbf{R} \rightarrow [-1, 1]$
- Tangent function, i.e., tan : $\mathbf{R} - \{ x : x = (2n + 1)\pi/2 , n \in \mathbf{Z}\} \rightarrow \mathbf{R}$
- Cotangent function, i.e., cot : $\mathbf{R} - \{ x : x = n\pi, n \in \mathbf{Z}\} \rightarrow \mathbf{R}$
- Secant function, i.e., sec : $\mathbf{R} - \{ x : x = (2n + 1)\pi/2 , n \in \mathbf{Z}\} \rightarrow \mathbf{R} - (-1, 1)$
- Cosecant function, i.e., cosec : $\mathbf{R} - \{ x : x = n\pi, n \in \mathbf{Z}\} \rightarrow \mathbf{R} - (-1, 1)$



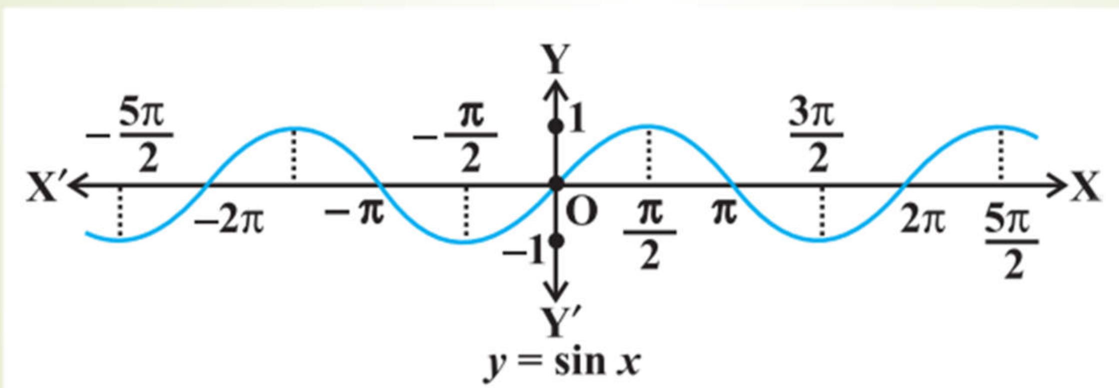
Inverse of Sin function

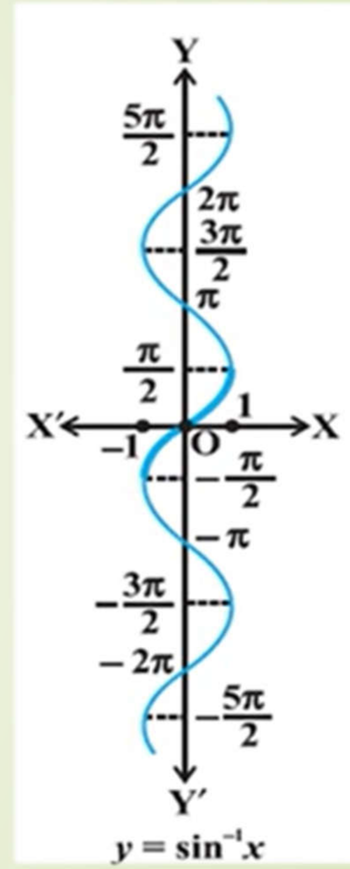
- Natural domain & range for Sine : R → [– 1, 1]
- If we restrict domain to [- π/2 , π/2], then it becomes one-one & onto with range [– 1, 1]
- Restricted domain & range of sine: [-π/2, π/2] → [– 1, 1]
- Restricted domain & range of Sin^{-1} : [– 1, 1] →[-π/2, π/2]
- [-π/2, π/2] is called the *principal value branch*
- *If* $y = \mathrm{Sin}^{-1} x$, $\sin y = x$



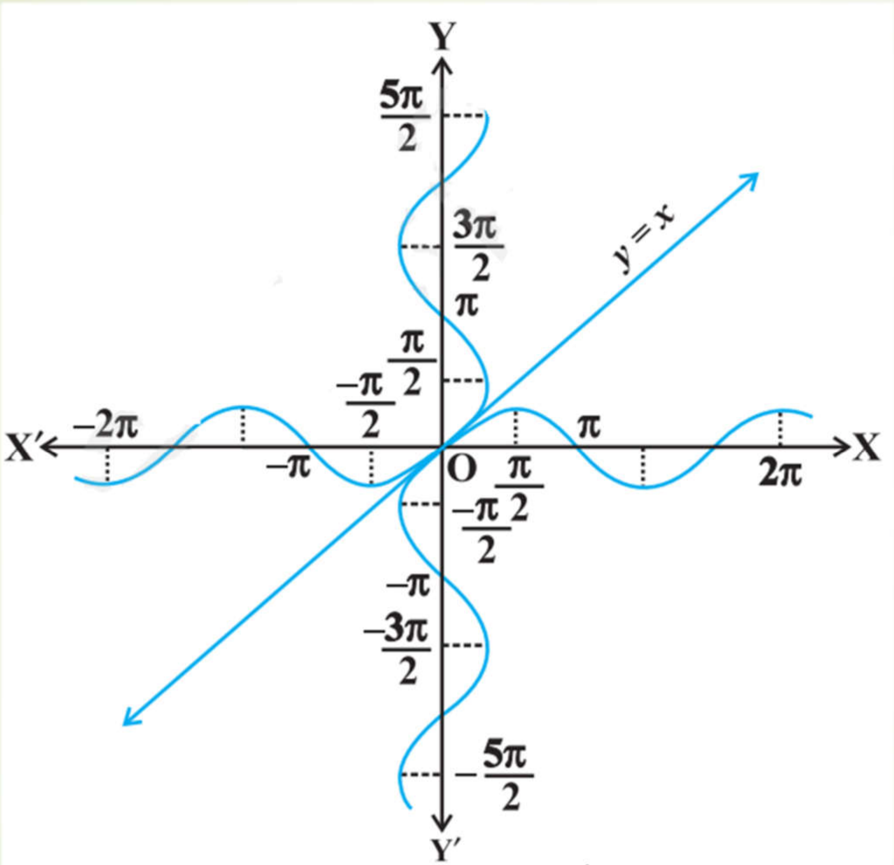
Graph for Sin^{-1} x

- The graph of Sin^{-1} function can be obtained from the graph of original function by interchanging x and y axes.
- It can be shown that the graph of an inverse function can be obtained from the corresponding graph of original function as a mirror image (i.e., reflection) along the line $y = x$.

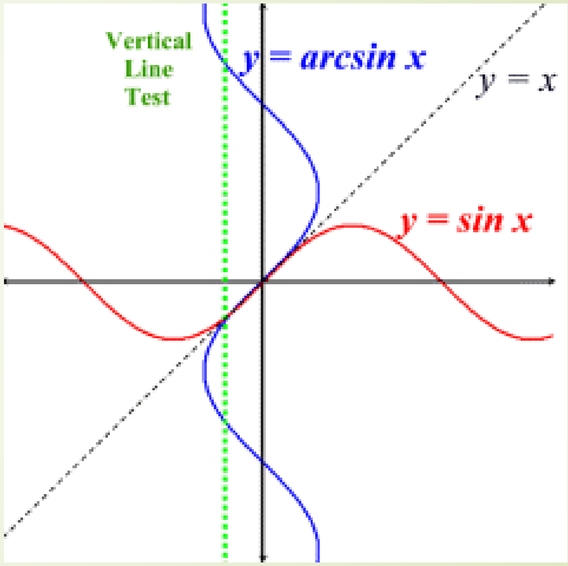




Mirror image

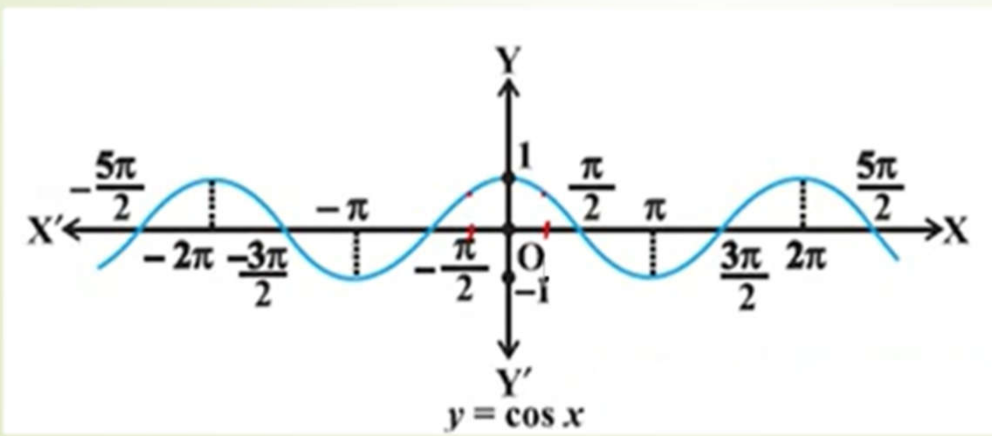


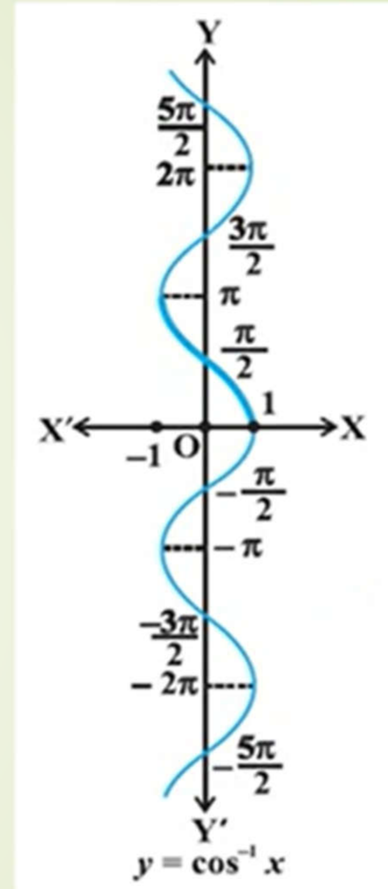
$y = \sin x$ and $y = \sin^{-1} x$



Inverse of Cos function

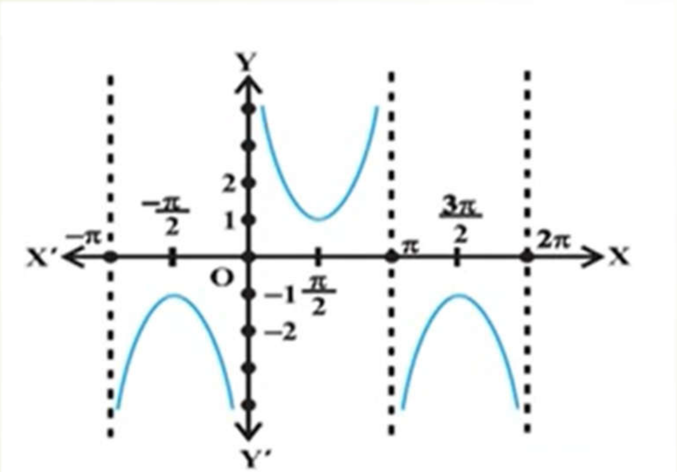
- Natural domain & range for Cos : R → [– 1, 1]
- If we restrict domain to [0 , π], then it becomes one-one & onto range [– 1, 1]
- Restricted domain & range of cosine: [0 , π] → [– 1, 1]
- Restricted domain & range of Cos^{-1} : [– 1, 1] →[0 , π]
- [0 , π] is called the *principal value branch.*



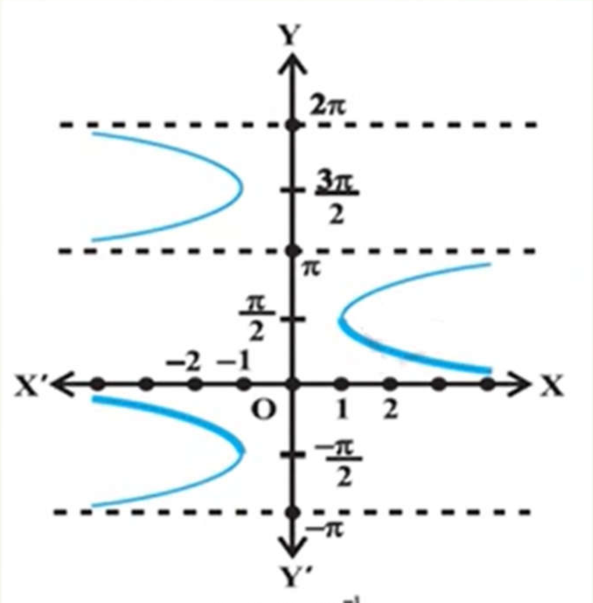


Inverse of Cosec function

- Natural domain & range for Cosec : R – { x : $x = n\pi$, $n \in$ Z} → R – (– 1, 1)
- If we restrict domain to [- π/2 , π/2] – {0}, then it becomes one-one & onto.
- Restricted domain & range of cosec : [- π/2 , π/2] – {0} → R - [– 1, 1]
- Restricted domain & range of Cosec^{-1} : R – (– 1, 1) → [-π/2, π/2] – {0}
- [-π/2, π/2] – {0} is called the *principal value branch.*



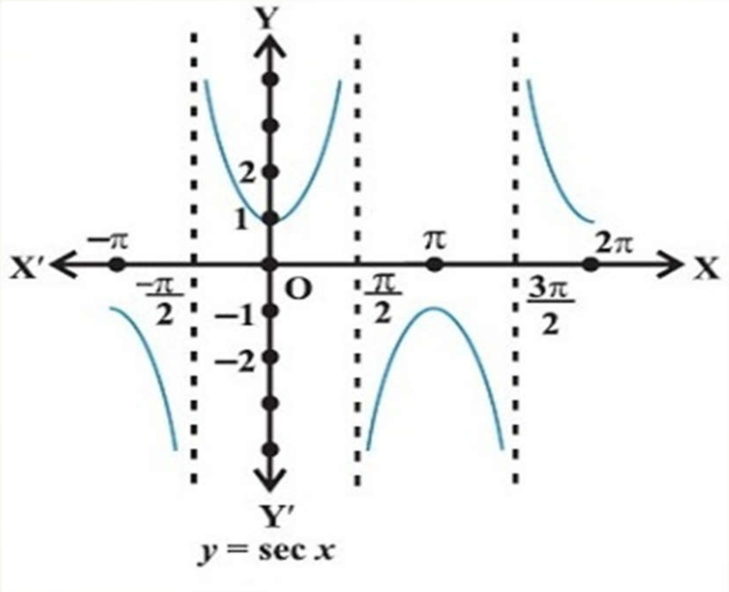
y = cosec x

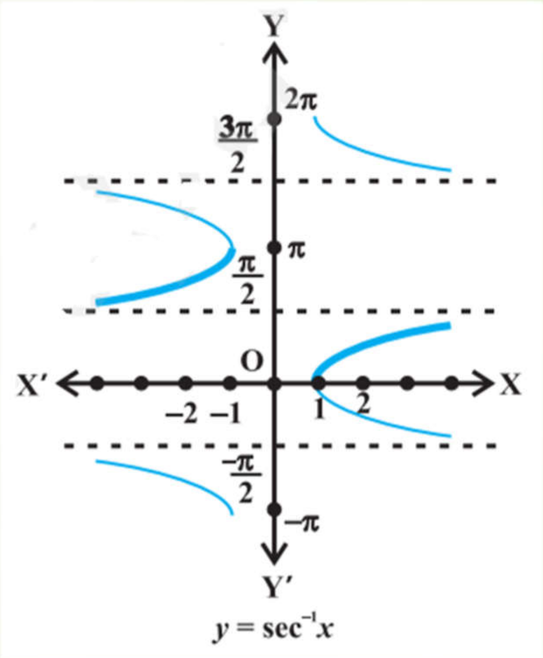


y = cosec⁻¹x

Inverse of Sec function

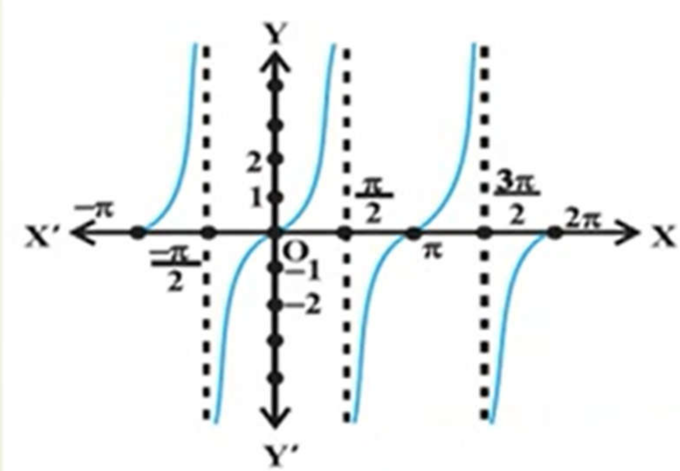
- Natural domain & range for Sec : R – { x : $x = n\pi$, $n \in$ Z} → R – (– 1, 1)
- If we restrict domain to [0, π] – {π/2}, then it becomes one-one & onto.
- Restricted domain & range of sec : [0, π] – {π/2}, → R – (– 1, 1)
- Restricted domain & range of Sec^{-1} : R – (– 1, 1) → [0, π] – {π/2}
- [0, π] – {π/2} is called the *principal value branch.*



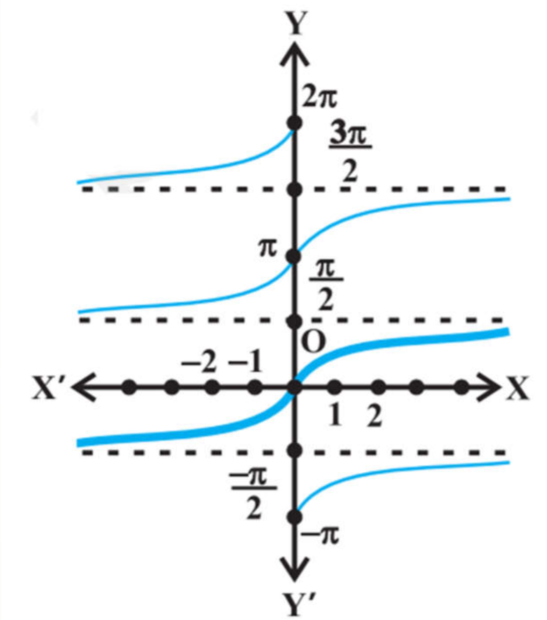


Inverse of Tan function

- Natural domain & range for Tan : $R - \{ x : x = (2n + 1)\pi/2 \, , n \in Z\} \to R$
- If we restrict domain to $(-\pi/2, \pi/2)$, then it becomes one-one & onto.
- Restricted domain & range of tan : $(-\pi/2, \pi/2) \to R$
- Restricted domain & range of $Tan^{-1} : R \to (-\pi/2, \pi/2)$
- $(-\pi/2, \pi/2)$ is called the *principal value branch*.



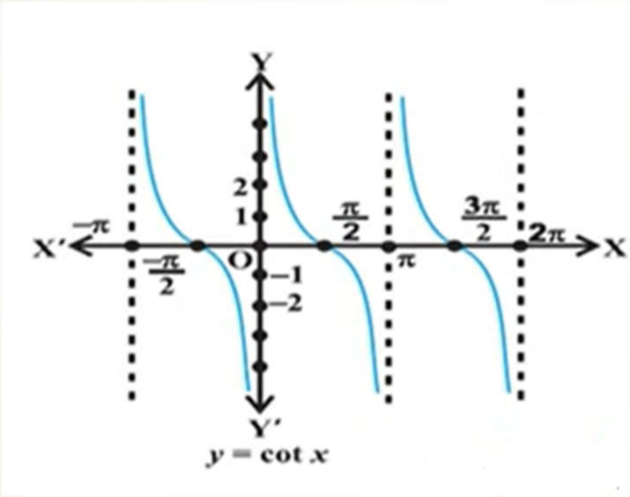
$y = \tan x$

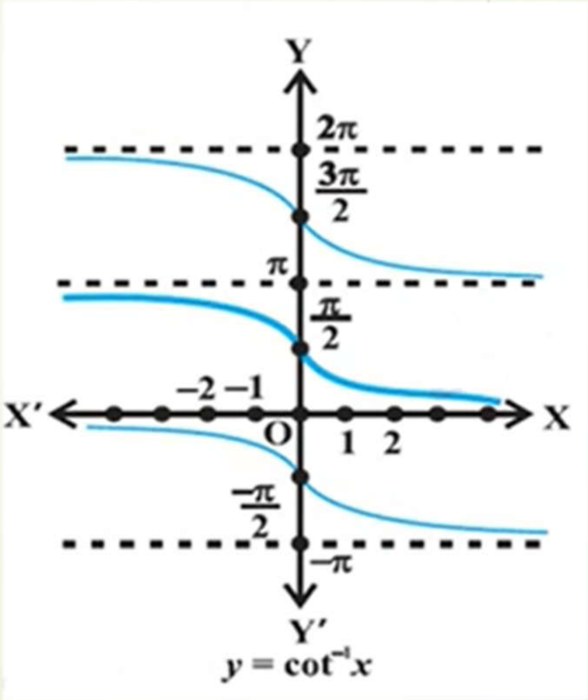


$y = \tan^{-1} x$

Inverse of Cot function

- Natural domain & range for cot : R – { x : x = nπ, n ∈ Z} → R
- If we restrict domain to (0, π) , then it becomes one-one & onto with range R.
- Restricted domain & range of cot : (0, π) → R
- Restricted domain & range of Cot^{-1} : R → (0, π)
- (0, π) is called the *principal value branch.*





Note

1. $\sin^{-1}x$ should not be confused with $(\sin x)^{-1}$. In fact $(\sin x)^{-1} = 1/\sin x$ and similarly for other trigonometric functions.
2. Whenever no branch of an inverse trigonometric functions is mentioned, we mean the principal value branch of that function.
3. The value of an inverse trigonometric functions which lies in the range of principal branch is called the *principal value* of that inverse trigonometric functions.

$\sin^{-1}$	:	$[-1, 1]$	$\rightarrow$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}$	:	$[-1, 1]$	$\rightarrow$	$[0, \pi]$
cosec^{-1}	:	$\mathbf{R} - (-1,1)$	$\rightarrow$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1}$	:	$\mathbf{R} - (-1, 1)$	$\rightarrow$	$[0, \pi] - \{\frac{\pi}{2}\}$
$\tan^{-1}$	:	$\mathbf{R}$	$\rightarrow$	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}$	:	$\mathbf{R}$	$\rightarrow$	$(0, \pi)$

Properties of Inverse Trigonometric Functions

1.
I. $\sin^{-1}(1/x) = \text{cosec}^{-1} x$; $x \geq 1$ or $x \leq -1$

II. $\cos^{-1}(1/x) = \sec^{-1} x$; $x \geq 1$ or $x \leq -1$

III. $\tan^{-1}(1/x) = \cot^{-1} x$; $x > 0$

2.
I. $\sin^{-1}(-x) = -\sin^{-1} x$; x ε $[-1, 1]$

II. $\tan^{-1}(-x) = -\tan^{-1} x$; x ε R

III. $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x$; modulus $x \geq 1$

Cont.

3.
I. $\cos^{-1}(-x) = \pi - \cos^{-1} x$; $x \varepsilon [-1,1]$

II. $\sec^{-1}(-x) = \pi - \sec^{-1} x$; modulus $x \geq 1$

III. $\cot^{-1}(-x) = \pi - \cot^{-1} x$; if $x \varepsilon$ 'R'

4.
I. $\sin^{-1} x + \cos^{-1} x = \pi/2$; if $x \varepsilon [-1,1]$

II. $\tan^{-1} x + \cot^{-1} x = \pi/2$; if $x \varepsilon$ 'R'

III. $\mathrm{cosec}^{-1} x + \sec^{-1} x = \pi/2$; if modulus $x \geq 1$

Cont.

5. I. $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x + y}{1 - x.y}\right)$, $xy < 1$

II. $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x - y}{1 + x.y}\right)$, $xy > -1$

6. I. $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$, modulus $x \leq 1$

II. $2\tan^{-1} x = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$, $x \geq 0$

III. $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$, $-1 < x < 1$

☺